Exercise 39

Find the limit or show that it does not exist.

$$\lim_{x\to\infty}(e^{-2x}\cos x)$$

Solution

Make the substitution, $x = 2u$. Then as $x \to \infty$, $u \to \infty$.

$$\begin{aligned}
\lim_{x\to\infty} e^{-2x}\cos x &= \lim_{u\to\infty} e^{-2(2u)}\cos 2u \\
&= \lim_{u\to\infty} e^{-4u}\cos 2u \\
&= \lim_{u\to\infty} e^{-4u}(1-2\sin^2 u) \\
&= \lim_{u\to\infty} (e^{-4u}-2e^{-4u}\sin^2 u) \\
&= \lim_{u\to\infty} e^{-4u} - \lim_{u\to\infty} 2e^{-4u}\sin^2 u \\
&= 0 - 2\lim_{u\to\infty} e^{-4u}\sin^2 u \\
&= -2\lim_{u\to\infty} u^2 e^{-4u}\left(\frac{\sin u}{u}\right)\left(\frac{\sin u}{u}\right) \\
&= -2\left(\lim_{u\to\infty} u^2 e^{-4u}\right)\left(\lim_{u\to\infty}\frac{\sin u}{u}\right)\left(\lim_{u\to\infty}\frac{\sin u}{u}\right) \\
&= -2(0)(0)(0) \\
&= 0
\end{aligned}$$